W3C Working Draft

OWL 2 Web Ontology Language: Direct Semantics

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Abstract

OWL 2 extends the W3C OWL Web Ontology Language with a small but useful set of features that have been requested by users, for which effective reasoning algorithms are now available, and that OWL tool developers are willing to support. The new features include extra syntactic sugar, additional property and qualified cardinality constructors, extended datatype support, simple metamodeling, and extended annotations.
This document provides the direct model-theoretic semantics for OWL 2, which is compatible with the description logic *SROIQ*. Furthermore, this document defines the most common inference problems for OWL 2.

Status of this Document

May Be Superseded

This section describes the status of this document at the time of its publication. Other documents may supersede this document. A list of current W3C publications and the latest revision of this technical report can be found in the W3C technical reports index at http://www.w3.org/TR/.

Set of Documents

This document is being published as one of a set of 11 documents:

1. Structural Specification and Functional-Style Syntax
2. Direct Semantics (this document)
3. RDF-Based Semantics
4. Conformance and Test Cases
5. Mapping to RDF Graphs
6. XML Serialization
7. Profiles
8. Quick Reference Guide
9. New Features and Rationale
10. Manchester Syntax
11. rdf:text: A Datatype for Internationalized Text

Last Call

The Working Group believes it has completed its design work for the technologies specified this document, so this is a "Last Call" draft. The design is not expected to change significantly, going forward, and now is the key time for external review, before the implementation phase.

Summary of Changes

This document has been updated to keep in sync with the Syntax document. The most significant update is in the formal definition of the datatype map.

Please Comment By 23 January 2009

The OWL Working Group seeks public feedback on these Working Drafts. Please send your comments to public-owl-comments@w3.org (public archive). If possible, please offer specific changes to the text that would address your concern. You may also wish to check the Wiki Version of this document for internal-review comments and changes being drafted which may address your concerns.

No Endorsement

Patents

Contents

1 Introduction

This document defines the direct model-theoretic semantics of OWL 2. The semantics given here is strongly related to the semantics of description logics

[Description Logics] and is compatible with the semantics of the description logic *SROIQ* [SROIQ]. As the definition of *SROIQ* does not provide for datatypes and punning, the semantics of OWL 2 is defined directly on the constructs of the structural specification of OWL 2 [OWL 2 Specification] instead of by reference to *SROIQ*. For the constructs available in *SROIQ*, the semantics of *SROIQ* trivially corresponds to the one defined in this document.

Since OWL 2 is an extension of OWL DL, this document also provides a direct semantics for OWL Lite and OWL DL; this semantics is equivalent to the official semantics of OWL Lite and OWL DL [OWL Abstract Syntax and Semantics]. Furthermore, this document also provides the direct model-theoretic semantics for the OWL 2 profiles [OWL 2 Profiles].

The semantics is defined for an OWL 2 axioms and ontologies, which should be understood as instances of the structural specification [OWL 2 Specification]. Parts of the structural specification are written in this document using the functional-style syntax.

OWL 2 allows for annotations of ontologies, anonymous individuals, axioms, and other annotations. Annotations of all these types, however, have no semantic meaning in OWL 2 and are ignored in this document. OWL 2 declarations are used only to disambiguate class expressions from data ranges and object property from data property expressions in the functional-style syntax; therefore, they are not mentioned explicitly in this document.

2 Direct Model-Theoretic Semantics for OWL 2

This section specifies the direct model-theoretic semantics of OWL 2 ontologies.

2.1 Vocabulary

A *datatype map* is a 6-tuple $D = (N_{DT} , N_{LS} , N_{FS} , \cdot^{DT} , \cdot^{LS} , \cdot^{FS})$ with the following components.

- N_{DT} is a set of datatypes that does not contain the datatype *rdfs:Literal*.
- N_{LS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{LS}(DT)$ of strings called *lexical values*. The set $N_{LS}(DT)$ is called the *lexical space* of *DT*.
- N_{FS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{FS}(DT)$ of pairs $\langle F\ v \rangle$, where *F* is a *constraining facet* and *v* is an arbitrary object called a *value*. The set $N_{FS}(DT)$ is called the *facet space* of *DT*.
- For each datatype $DT \in N_{DT}$, the *interpretation function* $\cdot^{DT}$ assigns to *DT* a set $(DT)^{DT}$ called the *value space* of *DT*.
- For each datatype $DT \in N_{DT}$ and each lexical value $LV \in N_{LS}(DT)$, the *interpretation function* $\cdot^{LS}$ assigns to the pair $\langle LV\ DT \rangle$ a *data value* $(\langle LV\ DT \rangle)^{LS} \in (DT)^{DT}$.

- For each datatype $DT \in N_{DT}$ and each pair $\langle F\ v \rangle \in N_{FS}(DT)$, the *interpretation function* $\cdot^{FS}$ assigns to $\langle F\ v \rangle$ a *facet value* $(\langle F\ v \rangle)^{FS} \subseteq (DT)^{DT}$.

A *vocabulary* $V = (V_C , V_{OP} , V_{DP} , V_I , V_{DT} , V_{LT} , V_{FA})$ over a datatype map D is a 7-tuple consisting of the following elements:

- V_C is a set of *classes* as defined in the OWL 2 Specification [OWL 2 Specification], containing at least the classes *owl:Thing* and *owl:Nothing*.
- V_{OP} is a set of *object properties* as defined in the OWL 2 Specification [OWL 2 Specification], containing at least the object properties *owl:topObjectProperty* and *owl:bottomObjectProperty*.
- V_{DP} is a set of *data properties* as defined in the OWL 2 Specification [OWL 2 Specification], containing at least the data properties *owl:topDataProperty* and *owl:bttomDataProperty*.
- V_I is a set of *individuals* (named and anonymous) as defined in the OWL 2 Specification [OWL 2 Specification].
- V_{DT} is the set of all datatypes of D extended with the datatype *rdfs:Literal*; that is, $V_{DT} = N_{DT} \cup \{ \mathit{rdfs{:}Literal} \}$.
- V_{LT} is a set of *literals* $LV\hat{}\hat{}DT$ for each datatype $DT \in N_{DT}$ and each lexical value $LV \in N_{LS}(DT)$.
- V_{FA} is the set of pairs $\langle F\ lt \rangle$ for each constraining facet F, datatype $DT \in N_{DT}$, and literal $lt \in V_{LT}$ such that $\langle F\ (\langle LV\ DT_1 \rangle)^{LS} \rangle \in N_{FS}(DT)$, where LV is the lexical value of lt and DT_1 is the datatype of lt.

Given a vocabulary V, the following conventions are used in this document to denote different syntactic parts of OWL 2 ontologies:

- `OP` denotes an object property;
- `OPE` denotes an object property expression;
- `DP` denotes a data property;
- `DPE` denotes a data property expression;
- `PE` denotes an object property or a data property expression;
- `C` denotes a class;
- `CE` denotes a class expression;
- `DT` denotes a datatype;
- `DR` denotes a data range;
- `a` denotes an individual (named or anonymous);
- `lt` denotes a literal; and
- `F` denotes a constraining facet.

2.2 Interpretations

Given a datatype map D and a vocabulary V over D, an *interpretation* $Int = (\Delta_{Int} , \Delta_D , \cdot^C , \cdot^{OP} , \cdot^{DP} , \cdot^I , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ for D and V is a 9-tuple with the following structure.

- Δ_{Int} is a nonempty set called the *object domain*.

- Δ_D is a nonempty set disjoint with Δ_{Int} called the *data domain* such that $(DT)^{DT} \subseteq \Delta_D$ for each datatype $DT \in V_{DT}$.
- $\cdot^{C}$ is the *class interpretation* function that assigns to each class $C \in V_C$ a subset $(C)^{C} \subseteq \Delta_{Int}$ such that
  - $(owl{:}Thing)^{C} = \Delta_{Int}$ and
  - $(owl{:}Nothing)^{C} = \emptyset$.
- $\cdot^{OP}$ is the *object property interpretation* function that assigns to each object property $OP \in V_{OP}$ a subset $(OP)^{OP} \subseteq \Delta_{Int} \times \Delta_{Int}$ such that
  - $(owl{:}topObjectProperty)^{OP} = \Delta_{Int} \times \Delta_{Int}$ and
  - $(owl{:}bottomObjectProperty)^{OP} = \emptyset$.
- $\cdot^{DP}$ is the *data property interpretation* function that assigns to each data property $DP \in V_{DP}$ a subset $(DP)^{DP} \subseteq \Delta_{Int} \times \Delta_D$ such that
  - $(owl{:}topDataProperty)^{DP} = \Delta_{Int} \times \Delta_D$ and
  - $(owl{:}bottomDataProperty)^{DP} = \emptyset$.
- $\cdot^{I}$ is the *individual interpretation* function that assigns to each individual $a \in V_I$ an element $(a)^{I} \in \Delta_{Int}$.
- $\cdot^{DT}$ is the *datatype interpretation function* that is the same as in D for all datatypes $DT \in N_{DT}$ and is extended to *rdfsLiteral* by setting
  - $(rdfs{:}Literal)^{DT} = \Delta_D$.
- $\cdot^{LT}$ is the *literal interpretation function* that is defined as $(lt)^{LT} = (\langle LV\ DT \rangle)^{LS}$ for each $lt \in V_{LT}$, where LV is the lexical value of lt and DT is the datatype of lt.
- $\cdot^{FA}$ is the *facet interpretation function* that is defined as $(\langle F\ lt \rangle)^{FA} = (\langle F\ (lt)^{LT} \rangle)^{FS}$ for each $\langle F\ lt \rangle \in V_{FA}$.

The following sections define the extensions of $\cdot^{OP}$, $\cdot^{DT}$, and $\cdot^{C}$ to object property expressions, data ranges, and class expressions.

2.2.1 Object Property Expressions

The object property interpretation function $\cdot^{OP}$ is extended to object property expressions as shown in Table 1.

Table 1. Interpreting Object Property Expressions

Object Property Expression	Interpretation $\cdot^{OP}$
`InverseOf( OP )`	$\{ \langle x , y \rangle \mid \langle y , x \rangle \in (OP)^{OP} \}$

2.2.2 Data Ranges

The datatype interpretation function $\cdot^{DT}$ is extended to data ranges as shown in Table 3. All datatypes in OWL 2 are unary, so each datatype DT is interpreted as a unary relation over Δ_D — that is, a set $(DT)^{DT} \subseteq \Delta_D$. Data ranges, however, can be n-ary, as this allows implementations to extend OWL 2 with built-in operations such as comparisons or arithmetic. An n-ary data range DR is interpreted as an n-ary relation $(DR)^{DT}$ over Δ_D.

Table 3. Interpreting Data Ranges

Data Range	**Interpretation** $\cdot^{DT}$
IntersectionOf(DR_1 ... DR_n)	$(DR_1)^{DT} \cap \ldots \cap (DR_n)^{DT}$
UnionOf(DR_1 ... DR_n)	$(DR_1)^{DT} \cup \ldots \cup (DR_n)^{DT}$
ComplementOf(DR)	$(\Delta_D)^n \setminus (DR)^{DT}$ where n is the arity of DR
OneOf(lt_1 ... lt_n)	$\{ (lt_1)^{LT} , \ldots , (lt_n)^{LT} \}$
DatatypeRestriction(DT F_1 lt_1 ... F_n lt_n)	$(DT)^{DT} \cap (\langle F_1\ lt_1 \rangle)^{FA} \cap \ldots \cap (\langle F_n\ lt_n \rangle)^{FA}$

2.2.3 Class Expressions

The class interpretation function $\cdot^C$ is extended to class expressions as shown in Table 4. For *S* a set, *#S* denotes the number of elements in *S*.

Table 4. Interpreting Class Expressions

Class Expression	**Interpretation** $\cdot^C$
IntersectionOf(CE_1 ... CE_n)	$(CE_1)^C \cap \ldots \cap (CE_n)^C$
UnionOf(CE_1 ... CE_n)	$(CE_1)^C \cup \ldots \cup (CE_n)^C$
ComplementOf(CE)	$\Delta_{Int} \setminus (CE)^C$
OneOf(a_1 ... a_n)	$\{ (a_1)^I , \ldots , (a_n)^I \}$
SomeValuesFrom(OPE CE)	$\{ x \mid \exists\ y : \langle x, y \rangle \in (OPE)^{OP}$ and $y \in (CE)^C \}$
AllValuesFrom(OPE CE)	$\{ x \mid \forall\ y : \langle x, y \rangle \in (OPE)^{OP}$ implies $y \in (CE)^C \}$
HasValue(OPE a)	$\{ x \mid \langle x , (a)^I \rangle \in (OPE)^{OP} \}$
HasSelf(OPE)	$\{ x \mid \langle x , x \rangle \in (OPE)^{OP} \}$
MinCardinality(n OPE)	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP} \} \geq n \}$
MaxCardinality(n OPE)	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP} \} \leq n \}$

`ExactCardinality( n OPE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP} \} = n \}$
`MinCardinality( n OPE CE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP}$ and $y \in (CE)^{C} \} \geq n \}$
`MaxCardinality( n OPE CE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP}$ and $y \in (CE)^{C} \} \leq n \}$
`ExactCardinality( n OPE CE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (OPE)^{OP}$ and $y \in (CE)^{C} \} = n \}$
`SomeValuesFrom( DPE`$_1$` ... DPE`$_n$` DR )`	$\{ x \mid \exists\, y_1, \ldots , y_n : \langle x , y_k \rangle \in (DPE_k)^{DP}$ for each $1 \leq k \leq n$ and $\langle y_1 , \ldots , y_n \rangle \in (DR)^{DT} \}$
`AllValuesFrom( DPE`$_1$` ... DPE`$_n$` DR )`	$\{ x \mid \forall\, y_1, \ldots , y_n : \langle x , y_k \rangle \in (DPE_k)^{DP}$ for each $1 \leq k \leq n$ imply $\langle y_1 , \ldots , y_n \rangle \in (DR)^{DT} \}$
`HasValue( DPE lt )`	$\{ x \mid \langle x , (lt)^{LT} \rangle \in (DPE)^{DP} \}$
`MinCardinality( n DPE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP} \} \geq n \}$
`MaxCardinality( n DPE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP} \} \leq n \}$
`ExactCardinality( n DPE )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP} \} = n \}$
`MinCardinality( n DPE DR )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP}$ and $y \in (DR)^{DT} \} \geq n \}$
`MaxCardinality( n DPE DR )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP}$ and $y \in (DR)^{DT} \} \leq n \}$
`ExactCardinality( n DPE DR )`	$\{ x \mid \#\{ y \mid \langle x , y \rangle \in (DPE)^{DP}$ and $y \in (DR)^{DT} \} = n \}$

2.3 Satisfaction in an Interpretation

An interpretation $Int = (\Delta_{Int} , \Delta_D , \cdot^{C} , \cdot^{OP} , \cdot^{DP} , \cdot^{I} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ *satisfies* an axiom w.r.t. an ontology *O* if the axiom satisfies appropriate conditions listed in the following sections. Satisfaction of axioms in *Int* is defined w.r.t. *O* because satisfaction of key axioms uses the following function:

> $ISNAMED_O(x) = true$ for $x \in \Delta_{Int}$ if and only if $(a)^I = x$ for some named individual *a* occurring in the axiom closure of *O*

2.3.1 Class Expression Axioms

Satisfaction of OWL 2 class expression axioms in *Int* w.r.t. *O* is defined as shown in Table 5.

Table 5. Satisfaction of Class Expression Axioms in an Interpretation

Axiom	Condition
SubClassOf(CE_1 CE_2)	$(CE_1)^C \subseteq (CE_2)^C$
EquivalentClasses(CE_1 ... CE_n)	$(CE_j)^C = (CE_k)^C$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointClasses(CE_1 ... CE_n)	$(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
DisjointUnion(C CE_1 ... CE_n)	$(C)^C = (CE_1)^C \cup \ldots \cup (CE_n)^C$ and $(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

2.3.2 Object Property Expression Axioms

Satisfaction of OWL 2 object property expression axioms in *Int* w.r.t. *O* is defined as shown in Table 6.

Table 6. Satisfaction of Object Property Expression Axioms in an Interpretation

Axiom	Condition
SubPropertyOf(OPE_1 OPE_2)	$(OPE_1)^{OP} \subseteq (OPE_2)^{OP}$
SubPropertyOf(PropertyChain(OPE_1 ... OPE_n) OPE)	$\forall y_0 , \ldots , y_n : \langle y_0 , y_1 \rangle \in (OPE_1)^{OP}$ and ... and $\langle y_{n-1} , y_n \rangle \in (OPE_n)^{OP}$ imply $\langle y_0 , y_n \rangle \in (OPE)^{OP}$
EquivalentProperties(OPE_1 ... OPE_n)	$(OPE_j)^{OP} = (OPE_k)^{OP}$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointProperties(OPE_1 ... OPE_n)	$(OPE_j)^{OP} \cap (OPE_k)^{OP} = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
PropertyDomain(OPE CE)	$\forall x , y : \langle x , y \rangle \in (OPE)^{OP}$ implies $x \in (CE)^C$
PropertyRange(OPE CE)	$\forall x , y : \langle x , y \rangle \in (OPE)^{OP}$ implies $y \in (CE)^C$

`InverseProperties( OPE`$_1$ `OPE`$_2$ `)`	$(OPE_1)^{OP} = \{ \langle x , y \rangle \mid \langle y , x \rangle \in (OPE_2)^{OP} \}$
`FunctionalProperty( OPE )`	$\forall x , y_1 , y_2 : \langle x , y_1 \rangle \in (OPE)^{OP}$ and $\langle x , y_2 \rangle \in (OPE)^{OP}$ imply $y_1 = y_2$
`InverseFunctionalProperty( OPE )`	$\forall x_1 , x_2 , y : \langle x_1 , y \rangle \in (OPE)^{OP}$ and $\langle x_2 , y \rangle \in (OPE)^{OP}$ imply $x_1 = x_2$
`ReflexiveProperty( OPE )`	$\forall x : x \in \Delta_{Int}$ implies $\langle x , x \rangle \in (OPE)^{OP}$
`IrreflexiveProperty( OPE )`	$\forall x : x \in \Delta_{Int}$ implies $\langle x , x \rangle \notin (OPE)^{OP}$
`SymmetricProperty( OPE )`	$\forall x , y : \langle x , y \rangle \in (OPE)^{OP}$ implies $\langle y , x \rangle \in (OPE)^{OP}$
`AsymmetricProperty( OPE )`	$\forall x , y : \langle x , y \rangle \in (OPE)^{OP}$ implies $\langle y , x \rangle \notin (OPE)^{OP}$
`TransitiveProperty( OPE )`	$\forall x , y , z : \langle x , y \rangle \in (OPE)^{OP}$ and $\langle y , z \rangle \in (OPE)^{OP}$ imply $\langle x , z \rangle \in (OPE)^{OP}$

2.3.3 Data Property Expression Axioms

Satisfaction of OWL 2 data property expression axioms in *Int* w.r.t. *O* is defined as shown in Table 7.

Table 7. Satisfaction of Data Property Expression Axioms in an Interpretation

Axiom	Condition
`SubPropertyOf( DPE`$_1$ `DPE`$_2$ `)`	$(DPE_1)^{DP} \subseteq (DPE_2)^{DP}$
`EquivalentProperties( DPE`$_1$ `... DPE`$_n$ `)`	$(DPE_j)^{DP} = (DPE_k)^{DP}$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
`DisjointProperties( DPE`$_1$ `... DPE`$_n$ `)`	$(DPE_j)^{DP} \cap (DPE_k)^{DP} = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
`PropertyDomain( DPE CE )`	$\forall x , y : \langle x , y \rangle \in (DPE)^{DP}$ implies $x \in (CE)^{C}$
`PropertyRange( DPE DR )`	$\forall x , y : \langle x , y \rangle \in (DPE)^{DP}$ implies $y \in (DR)^{DT}$
`FunctionalProperty( DPE )`	$\forall x , y_1 , y_2 : \langle x , y_1 \rangle \in (DPE)^{DP}$ and $\langle x , y_2 \rangle \in (DPE)^{DP}$ imply $y_1 = y_2$

2.3.4 Keys

Satisfaction of keys in *Int* w.r.t. *O* is defined as shown in Table 8.

Table 8. Satisfaction of Keys in an Interpretation

Axiom	Condition
HasKey(CE PE_1 ... PE_n)	$\forall x , y , z_1 , \ldots , z_n$: if $ISNAMED_O(x)$ and $ISNAMED_O(y)$ and $ISNAMED_O(z_1)$ and ... and $ISNAMED_O(z_n)$ and $x \in (CE)^C$ and $y \in (CE)^C$ and for each $1 \leq i \leq n$, if PE_i is an object property, then $\langle x , z_i \rangle \in (PE_i)^{OP}$ and $\langle y , z_i \rangle \in (PE_i)^{OP}$, and if PE_i is a data property, then $\langle x , z_i \rangle \in (PE_i)^{DP}$ and $\langle y , z_i \rangle \in (PE_i)^{DP}$ then x = y

2.3.5 Assertions

Satisfaction of OWL 2 assertions in *Int* w.r.t. *O* is defined as shown in Table 9.

Table 9. Satisfaction of Assertions in an Interpretation

Axiom	Condition
SameIndividual(a_1 ... a_n)	$(a_j)^I = (a_k)^I$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DifferentIndividuals(a_1 ... a_n)	$(a_j)^I \neq (a_k)^I$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
ClassAssertion(CE a)	$(a)^I \in (CE)^C$
PropertyAssertion(OPE a_1 a_2)	$\langle (a_1)^I , (a_2)^I \rangle \in (OPE)^{OP}$
NegativePropertyAssertion(OPE a_1 a_2)	$\langle (a_1)^I , (a_2)^I \rangle \notin (OPE)^{OP}$
PropertyAssertion(DPE a lt)	$\langle (a)^I , (lt)^{LT} \rangle \in (DPE)^{DP}$
NegativePropertyAssertion(DPE a lt)	$\langle (a)^I , (lt)^{LT} \rangle \notin (DPE)^{DP}$

2.3.6 Ontologies

Int satisfies an OWL 2 ontology *O* if all axioms in the axiom closure of *O* (with anonymous individuals renamed apart as described in Section 5.6.2 of the OWL 2 Specification [*OWL 2 Specification*]) are satisfied in *Int* w.r.t. *O*.

2.4 Models

An interpretation $Int = (\Delta_{Int} , \Delta_D , \cdot^C , \cdot^{OP} , \cdot^{DP} , \cdot^I , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ is a *model* of an OWL 2 ontology *O* if an interpretation $Int_1 = (\Delta_{Int} , \Delta_D , \cdot^C , \cdot^{OP} , \cdot^{DP} , \cdot^{I_1} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ exists such that $\cdot^{I_1}$ coincides with $\cdot^I$ on all named individuals and Int_1 satisfies *O*.

Thus, an interpretation *Int* satisfying *O* is also a model of *O*. In contrast, a model *Int* of *O* may not satisfy *O* directly; however, by modifying the interpretation of anonymous individuals, *Int* can always be coerced into an interpretation Int_1 that satisfies *O*.

2.5 Inference Problems

Let *D* be a datatype map and *V* a vocabulary over *D*. Furthermore, let *O* and O_1 be OWL 2 ontologies, *CE*, CE_1, and CE_2 class expressions, and *a* a named individual, such that all of them refer only to the vocabulary elements in *V*. A *Boolean conjunctive query* Q is a closed formula of the form

$$\exists\ x_1 , \ldots , x_n , y_1 , \ldots , y_m : [A_1 \wedge \ldots \wedge A_k]$$

where each A_i is an *atom* of the form `C(s)`, `OP(s,t)`, or `DP(s,u)` with `C` a class, `OP` an object property, `DP` a data property, `s` and `t` individuals or some variable x_j, and `u` a literal or some variable y_j.

The following inference problems are often considered in practice.

Ontology Consistency: *O* is *consistent* (or *satisfiable*) w.r.t. *D* if a model of *O* w.r.t. *D* and *V* exists.

Ontology Entailment: *O entails* O_1 w.r.t. *D* if every model of *O* w.r.t. *D* and *V* is also a model of O_1 w.r.t. *D* and *V*.

Ontology Equivalence: *O* and O_1 are *equivalent* w.r.t. *D* if *O* entails O_1 w.r.t. *D* and O_1 entails *O* w.r.t. *D*.

Ontology Equisatisfiability: *O* and O_1 are *equisatisfiable* w.r.t. *D* if *O* is satisfiable w.r.t. *D* if and only if O_1 is satisfiable w.r.t *D*.

Class Expression Satisfiability: CE is satisfiable w.r.t. O and D if a model $Int = (\Delta_{Int} , \Delta_D , \cdot^{C} , \cdot^{OP} , \cdot^{DP} , \cdot^{I} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ of O w.r.t. D and V exists such that $(CE)^C \neq \emptyset$.

Class Expression Subsumption: CE_1 is *subsumed* by a class expression CE_2 w.r.t. O and D if $(CE_1)^C \subseteq (CE_2)^C$ for each model $Int = (\Delta_{Int} , \Delta_D , \cdot^{C} , \cdot^{OP} , \cdot^{DP} , \cdot^{I} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ of O w.r.t. D and V.

Instance Checking: a is an *instance* of CE w.r.t. O and D if $(a)^I \in (CE)^C$ for each model $Int = (\Delta_{Int} , \Delta_D , \cdot^{C} , \cdot^{OP} , \cdot^{DP} , \cdot^{I} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ of O w.r.t. D and V.

Boolean Conjunctive Query Answering: Q is an *answer* w.r.t. O and D if Q is true in each model of O w.r.t. D and V.

In order to ensure that ontology entailment, class expression satisfiability, class expression subsumption, and instance checking are decidable, the following restriction w.r.t. O needs to be satisfied:

> Each class expression of type **MinObjectCardinality**, **MaxObjectCardinality**, **ExactObjectCardinality**, and **ObjectHasSelf** that occurs in O_1, CE, CE_1, and CE_2 can contain only object property expressions that are simple in the axiom closure Ax of O.

For ontology equivalence to be decidable, O_1 needs to satisfy this restriction w.r.t. O and vice versa. These restrictions are analogous to the first condition from Section 11.2 of the OWL 2 Specification [OWL 2 Specification].

3 Independence of the Semantics from the Datatype Map

The semantics of OWL 2 has been defined in such a way that the semantics of an OWL 2 ontology O does not depend on the choice of a datatype map, as long as the datatype map chosen contains all the datatypes occurring in O. This statement is made precise by the following theorem, which has several useful consequences:

- One can interpret an OWL 2 ontology O by considering only the datatypes explicitly occurring in O.
- When referring to various reasoning problems, the datatype map D need not be given explicitly, as it is sufficient to consider an implicit datatype map containing only the datatypes from the given ontology.
- OWL 2 reasoners can provide datatypes not explicitly mentioned in this specification without fear that this will change the semantics of OWL 2 ontologies not using these datatypes.

Theorem DS1. Let O_1 and O_2 be OWL 2 ontologies over a vocabulary V and $D = (N_{DT} , N_{LS} , N_{FS} , \cdot^{DT} , \cdot^{LS} , \cdot^{FS})$ a datatype map such that each datatype mentioned in O_1 and O_2 is either *rdfs:Literal* or it occurs in N_{DT}. Furthermore, let $D' = (N_{DT}' , N_{LS}' , N_{FS}' , \cdot^{DT\,'} , \cdot^{LS\,'} , \cdot^{FS\,'})$ be a datatype map such that $N_{DT} \subseteq N_{DT}'$, $N_{LS}(DT) = N_{LS}'(DT)$, and $N_{FS}(DT) = N_{FS}'(DT)$ for each $DT \in N_{DT}$, and $\cdot^{DT\,'}$,

$\cdot^{LS\,'}$, and $\cdot^{FS\,'}$ are extensions of $\cdot^{DT}$, $\cdot^{LS}$, and $\cdot^{FS}$, respectively. Then, O_1 entails O_2 w.r.t. D if and only if O_1 entails O_2 w.r.t. D'.

Proof. Without loss of generality, one can assume O_1 and O_2 to be in negation-normal form [Description Logics]. The claim of the theorem is equivalent to the following statement: an interpretation *Int* w.r.t. D and V exists such that O_1 is and O_2 is not satisfied in *Int* if and only if an interpretation *Int'* w.r.t. D' and V exists such that O_1 is and O_2 is not satisfied in *Int'*. The (⇐) direction is trivial since each interpretation *Int* w.r.t. D' and V is also an interpretation w.r.t. D and V. For the (⇒) direction, assume that an interpretation $Int = (\Delta_{Int} , \Delta_D , \cdot^{C} , \cdot^{OP} , \cdot^{DP} , \cdot^{I} , \cdot^{DT} , \cdot^{LT} , \cdot^{FA})$ w.r.t. D and V exists such that O_1 is and O_2 is not satisfied in *Int*. Let $Int' = (\Delta_{Int} , \Delta_D' , \cdot^{C\,'} , \cdot^{OP} , \cdot^{DP\,'} , \cdot^{I} , \cdot^{DT\,'} , \cdot^{LT\,'} , \cdot^{FA\,'})$ be an interpretation such that

- Δ_D' is obtained by extending Δ_D with the value space of all datatypes in $N_{DT}' \setminus N_{DT}$,
- $\cdot^{C\,'}$ coincides with $\cdot^{C}$ on all classes, and
- $\cdot^{DP\,'}$ coincides with $\cdot^{DP}$ on all data properties apart from *owl:topDataProperty*.

Clearly, $\texttt{ComplementOf(DR)}^{DT} \subseteq \texttt{ComplementOf(DR)}^{DT\,'}$ for each data range *DR* that is is either a datatype, a datatype restriction, or an enumerated data range. The *owl:topDataProperty* property can occur in O_1 and O_2 only in tautologies. The interpretation of all other data properties is the same in *Int* and *Int'*, so $(CE)^{C} = (CE)^{C\,'}$ for each class expression *CE* occurring in O_1 and O_2. Therefore, O_1 is and O_2 is not satisfied in *Int'*. QED

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